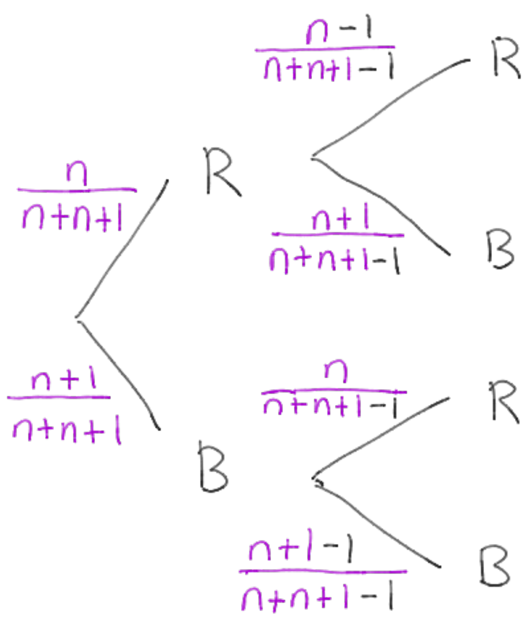
FULL MODEL ANSWERS

Q1. NON-CALCULATOR

There are only n red balls and $(n + 1)$ blue balls in a bag. Shamsa takes at random 2 balls from the bag.

Show that the probability that both balls are the same colour is $\dfrac{n}{2n+1}$



$P(R \text{ AND } R) = \dfrac{n}{2n+1} \times \dfrac{n-1}{2n} = \dfrac{n^2-n}{2n(2n+1)}$

$P(B \text{ AND } B) = \dfrac{n+1}{2n+1} \times \dfrac{n}{2n} = \dfrac{n^2+n}{2n(2n+1)}$

$P(\text{both same}) = \dfrac{n^2-n}{2n(2n+1)} + \dfrac{n^2+n}{2n(2n+1)}$

$= \dfrac{\cancel{2}n^{\cancel{2}}}{\cancel{2n}(2n+1)}$

$\dfrac{n}{2n+1}$

(Total for question = 4 marks)

Q2. NON-CALCULATOR

There are only r red counters and g green counters in a bag. A counter is taken at random from the bag.

The probability that the counter is green is $\dfrac{3}{7}$ ← assume this is simplified

The counter is put back in the bag.

2 more red counters and 3 more green counters are put in the bag. A counter is taken at random from the bag.

The probability that the counter is green is $\dfrac{6}{13}$ ← This could also be simplified (but it doesn't matter)

Find the number of red counters and the number of green counters that were in the bag originally.

Original P(Green): $\dfrac{3}{7} = \dfrac{6}{14} = \dfrac{9}{21} = \dfrac{12}{28} = \ldots$

New P(Green): $\dfrac{6}{13}$

Do any of the original fractions result in $\frac{6}{13}$ when 3 green and 2 red are added?

i.e. $\frac{3+3}{7+5} \neq \frac{6}{13}$ NO

$\frac{6+3}{14+5} \neq \frac{6}{13}$ NO

$\frac{9+3}{21+5} = \frac{6}{13}$ YES

Originally:

red counters ....12..........

green counters ....9..........

(Total for question = 5 marks)

Q3. NON-CALCULATOR

John has an empty box. He puts some red counters and some blue counters into the box.

The ratio of the number of red counters to the number of blue counters is 1 : 4

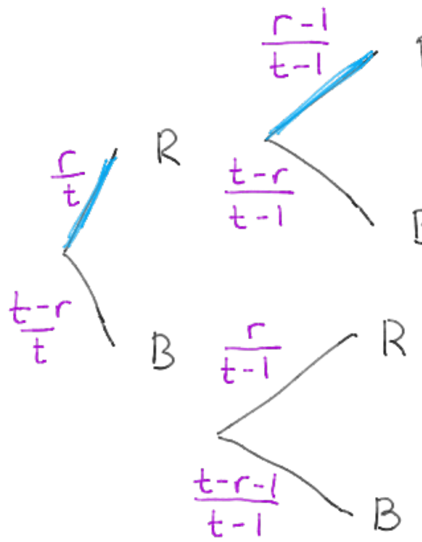
so red must be $\frac{1}{5}$ of the total

$r = \frac{t}{5}$

Linda takes at random 2 counters from the box.

The probability that she takes 2 red counters is $\frac{6}{155}$

How many red counters did John put into the box?



$R \rightarrow P(R \text{ AND } R) = \frac{r}{t} \times \frac{r-1}{t-1} = \frac{6}{155}$

$\frac{r(r-1)}{t(t-1)} = \frac{6}{155}$

$\frac{r^2-r}{t^2-t} = \frac{6}{155}$

$155r^2 - 155r = 6t^2 - 6t$

and since $t = 5r$

$155r^2 - 155r = 6(25r^2) - 6(5r)$

$155r^2 - 155r = 150r^2 - 30r$

$5r^2 = 125r$

$r^2 = 25r$

$r = 25$

..........25..........

(Total for question = 4 marks)

Q4. NON-CALCULATOR

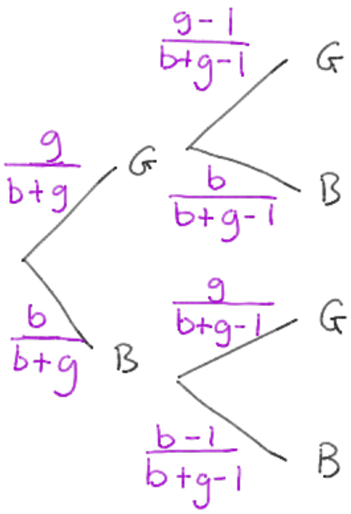
There are only green pens and blue pens in a box. $b-g=3 \longrightarrow b=g+3$

There are three more blue pens than green pens in the box. There are more than 12 pens in the box.

Simon is going to take at random two pens from the box.

The probability that Simon will take two pens of the same colour is $\frac{27}{55}$

Work out the number of green pens in the box.



$$P(G \text{ AND } G) = \frac{g}{b+g} \times \frac{g-1}{b+g-1} = \frac{g(g-1)}{(b+g)(b+g-1)}$$

$$P(B \text{ AND } B) = \frac{b}{b+g} \times \frac{b-1}{b+g-1} = \frac{b(b-1)}{(b+g)(b+g-1)}$$

$$P(\text{both same colour}) = P(G \text{ AND } G) + P(B \text{ AND } B)$$

$$\frac{27}{55} = \frac{g(g-1)}{(b+g)(b+g-1)} + \frac{b(b-1)}{(b+g)(b+g-1)}$$

substitute $b=g+3$

$$= \frac{g(g-1)}{(2g+3)(2g+2)} + \frac{(g+3)(g+2)}{(2g+3)(2g+2)}$$

cross multiply

$$\frac{27}{55} = \frac{g(g-1)+(g+3)(g+2)}{(2g+3)(2g+2)}$$

$$27(4g^2+10g+6) = 55(2g^2+4g+6)$$

$$108g^2+270g+162 = 110g^2+220g+330$$

$$0 = 2g^2-50g+168$$

factorise

$$0 = 2(g-21)(g-4)$$

$0=g-21$	$0=g-4$
$21=g$	$4=g$

"There are more than 12 pens in the box"

21

(Total for question = 6 marks)

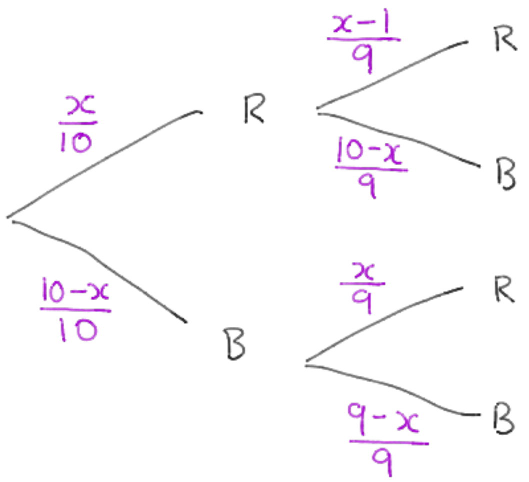
Q5. NON-CALCULATOR

There are 10 pens in a box.

There are x red pens in the box.
All the other pens are blue.

Jack takes at random two pens from the box.

Find an expression, in terms of x, for the probability that Jack takes one pen of each colour.
Give your answer in its simplest form.



$P(R \text{ AND } B) = \frac{x}{10} \times \frac{10-x}{9}$

$P(B \text{ AND } R) = \frac{10-x}{10} \times \frac{x}{9}$

$$P\left(\begin{matrix}\text{One of} \\ \text{each colour}\end{matrix}\right) = P(R \text{ AND } B) + P(B \text{ AND } R)$$

$$= \left(\frac{x}{10}\right) \times \left(\frac{10-x}{9}\right) + \left(\frac{10-x}{10}\right) \times \left(\frac{x}{9}\right)$$

$$= \frac{10x - x^2}{90} + \frac{10x - x^2}{90}$$

$$= \frac{10x - x^2 + 10x - x^2}{90}$$

$$= \frac{20x - 2x^2}{90}$$

$$= \frac{10x - x^2}{45}$$

$\frac{10x - x^2}{45}$ ....

(Total for question is 5 marks)

Q6. NON-CALCULATOR

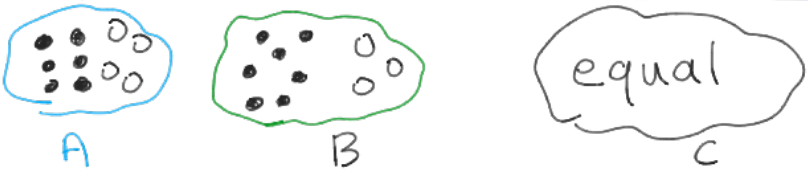
There are

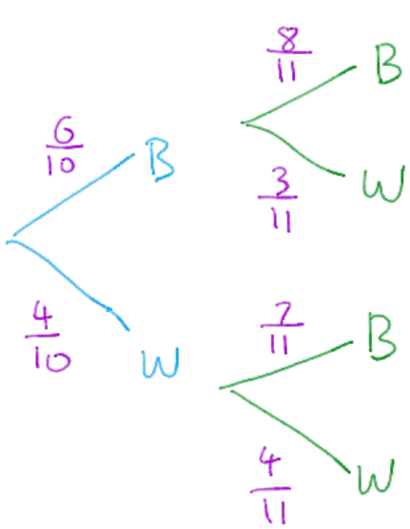
6 black counters and 4 white counters in bag **A**
7 black counters and 3 white counters in bag **B**
5 black counters and 5 white counters in bag **C**

A counter is not taken from bag C!

Bernie takes at random a counter from bag **A** and puts the counter in bag **B**.
He then takes at random a counter from bag **B** and puts the counter in bag **C**.

Find the probability that there are now more black counters than white counters in bag **C**.





$P(B \text{ AND } B) = \frac{6}{10} \times \frac{8}{11} = \frac{48}{110}$

$P(B \text{ AND } B) = \frac{4}{10} \times \frac{7}{11} = \frac{28}{110}$

$P\left(\begin{matrix}\text{More Black}\\ \text{than white}\end{matrix}\right) = P(BB) + P(BW) = \frac{38}{55}$

....

(Total for question = 3 marks)

Q7. CALCULATOR ALLOWED

There are 12 counters in a bag. $12 \div 3 = 4$
There is an equal number of red counters, blue counters and yellow counters in the bag.
There are no other counters in the bag.

3 counters are taken at random from the bag.

(a) Work out the probability of taking 3 red counters.

$P(RRR) = P(R) \times P(R) \times P(R)$

$= \frac{4}{12} \times \frac{3}{11} \times \frac{2}{10}$

$= \frac{24}{1320}$

$\frac{1}{55}$ ....

(2)

The 3 counters are put back into the bag.

Some more counters are now put into the bag.
There is still an equal number of red counters, blue counters and yellow counters in the bag.
There are no counters of any other colour in the bag.

3 counters are taken at random from the bag.

(b) Is it now less likely or equally likely or more likely that the 3 counters will be red? You must show how you get your answer.

It is now more likely, since the number of red counters is proportionally affected less after red counters are taken out. As an example, suppose there are 30 counters in total:

$$P(RRR) = \frac{10}{30} \times \frac{9}{29} \times \frac{8}{28} = \frac{720}{24360}$$

$$= \frac{6}{203} \qquad \frac{6}{203} > \frac{1}{55}$$

(2)

(Total for question = 4 marks)

Q8. CALCULATOR ALLOWED

Marek has 9 cards. There is a number on each card.

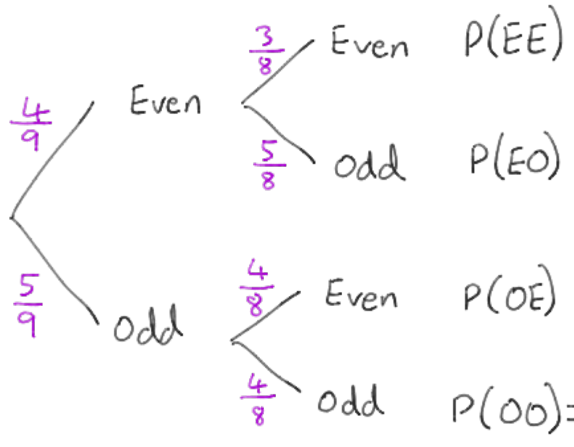
1	2	3	4	5	6	7	8	9
	even		even		even		even	

Marek takes at random two of the cards. He works out the product of the numbers on the two cards.

multiply

Work out the probability that the product is an even number.

odd × even = even
even × even = even
odd × odd = odd
even × odd = even



$$P(OO) = \frac{5}{9} \times \frac{4}{8} = \frac{20}{72}$$

$$1 - \frac{20}{72}$$

$$= \frac{13}{18}$$

....

(Total for question = 3 marks)

Q9. CALCULATOR ALLOWED

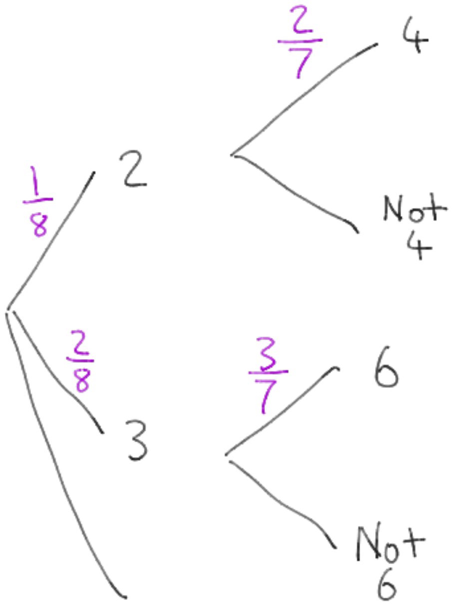
Here are 8 cards. There is a number on each card.

2	3	3	4	4	6	6	6

Erin puts the 8 cards in a bag.
She takes at random a card from the bag and does not replace it.
Erin then takes at random a second card from the bag.

2,4 OR 3,6

Calculate the probability that the number on the second card is double the number on the first card.



$P(2,4) = \frac{1}{8} \times \frac{2}{7} = \frac{2}{56}$

$P(3,6) = \frac{2}{8} \times \frac{3}{7} = \frac{6}{56}$

$\frac{2}{56} + \frac{6}{56}$

$\frac{8}{56}$

eeo eoe oee ooo

(Total for question = 3 marks)

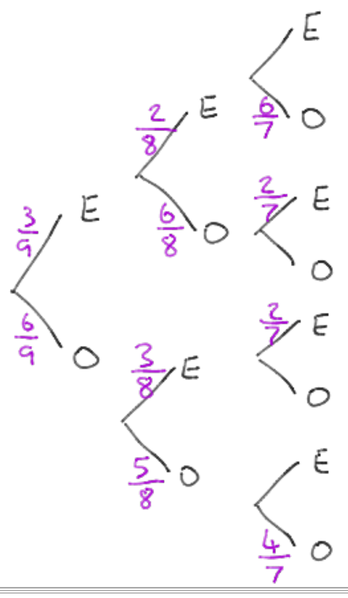
Q10. CALCULATOR ALLOWED

There are 9 counters in a bag. There is an even number on 3 of the counters. There is an odd number on 6 of the counters.

Three counters are going to be taken at random from the bag. The numbers on the counters will be added together to give the total.

Find the probability that the total is an odd number.

even + even + odd = odd
even + odd + even = odd
odd + even + even = odd
odd + odd + odd = odd



$P(EEO) = \frac{3}{9} \times \frac{2}{8} \times \frac{6}{7} = \frac{36}{504}$

$P(EOE) = \frac{3}{9} \times \frac{6}{8} \times \frac{2}{7} = \frac{36}{504}$

$P(OEE) = \frac{6}{9} \times \frac{3}{8} \times \frac{2}{7} = \frac{36}{504}$

$P(OOO) = \frac{6}{9} \times \frac{5}{8} \times \frac{4}{7} = \frac{120}{504}$

$$P(\text{odd total}) = \frac{36}{504} + \frac{36}{504} + \frac{36}{504} + \frac{120}{504}$$

$$= \frac{228}{504}$$

$\frac{19}{42}$

(Total for question = 5 marks)

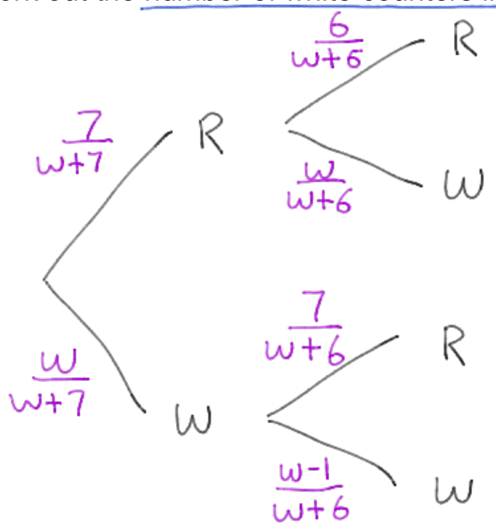
Q11. CALCULATOR ALLOWED

There are some red counters and some white counters in a bag.
At the start, 7 of the counters are red, the rest of the counters are white.

Alfie takes at random a counter from the bag. He does not put the counter back in the bag.
Alfie then takes at random another counter from the bag.

The probability that the first counter Alfie takes is white **and** the second counter Alfie takes is red is $\frac{21}{80}$

Work out the number of white counters in the bag at the start.



$$P(W, R) = \frac{w}{w+7} \times \frac{7}{w+6} = \frac{21}{80}$$

$$80 \times 7w = 21(w+7)(w+6)$$

$$80w = 3(w+7)(w+6)$$

$$80w = 3w^2 + 39w + 126$$

$$0 = 3w^2 - 41w + 126$$

$$0 = (3w-14)(w-9)$$

$0 = 3w - 14$ | $0 = w - 9$
$14 = 3w$ | $9 = w$

Since w must be an integer: $w = 9$

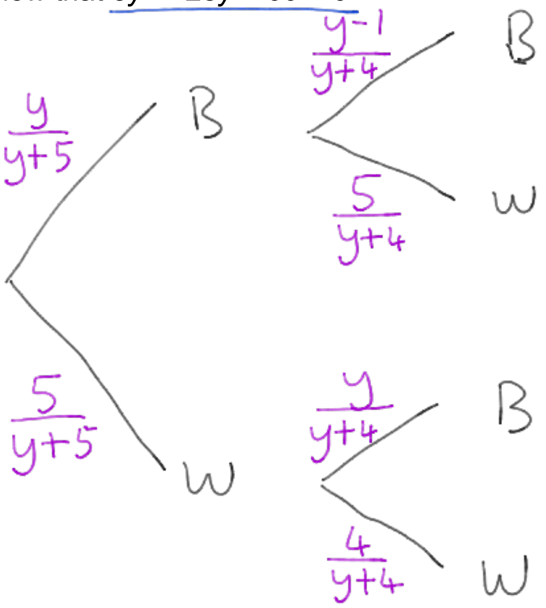
(Total for question = 5 marks)

Q12. CALCULATOR ALLOWED

There are y black socks and 5 white socks in a drawer. Joshua takes at random two socks from the drawer.

The probability that Joshua takes one white sock and one black sock is $\frac{6}{11}$

(a) Show that $3y^2 - 28y + 60 = 0$



$P(B,W) = \frac{y}{y+5} \times \frac{5}{y+4} = \frac{5y}{(y+5)(y+4)}$

$P(W,B) = \frac{5}{y+5} \times \frac{y}{y+4} = \frac{5y}{(y+5)(y+4)}$

$P\left(\begin{matrix}\text{BW in}\\ \text{any order}\end{matrix}\right) = \frac{6}{11} = 2 \times \frac{5y}{(y+5)(y+4)}$

$6(y+5)(y+4) = 110y$

$6y^2 + 54y + 120 = 110y$

$6y^2 - 56y + 120 = 0$

$3y^2 - 28y + 60 = 0$

(4)

(b) Find the probability that Joshua takes two black socks.

$3y^2 - 28y + 60 = 0$

$(3y-10)(y-6) = 0$

$3y - 10 = 0$	$y - 6 = 0$
$3y = 10$	
$y = \frac{10}{3}$	$y = 6$
NOT AN INTEGER	SOLUTION

$P(B,B) = \frac{6}{11} \times \frac{5}{10}$

$\frac{3}{11}$

(3)

(Total for question = 7 marks)